Problem 1.59

As you eat your way through a bag of chocolate chip cookies, you observe that each cookie is a circular disk with a diameter of 8.50 ± 0.02 cm and a thickness of 0.050 ± 0.005 cm. (a) Find the average volume of a cookie and the uncertainty in the volume. (b) Find the ratio of the diameter to the thickness and the uncertainty in this ratio.

Solution

Part (a)

The volume of this cylindrical cookie is

$$V = \pi r^2 h,$$

where

$$r = \frac{8.50 \pm 0.02 \text{ cm}}{2} = 4.25 \pm 0.01 \text{ cm}$$
$$h = 0.050 \pm 0.005 \text{ cm}.$$

Plugging in the numbers gives

$$V = \pi (4.25 \pm 0.01 \text{ cm})^2 (0.050 \pm 0.005 \text{ cm}).$$

The lower and upper bounds for the volume are

$$\text{Lower Bound}: \quad V = \pi (4.24 \text{ cm})^2 (0.045 \text{ cm}) \approx 2.5 \text{ cm}^3$$
$$\text{Upper Bound}: \quad V = \pi (4.26 \text{ cm})^2 (0.055 \text{ cm}) \approx 3.1 \text{ cm}^3.$$

Take the average of these two bounds.

$$\frac{2.5 + 3.1}{2} = 2.8$$

Both bounds can be covered with an uncertainty of 0.3. Therefore, the volume is

$$V = (2.8 \pm 0.3) \text{ cm}^3.$$

Part (b)

The ratio of the diameter to the thickness is

$$\frac{d}{h} = \frac{8.50 \pm 0.02 \text{ cm}}{0.050 \pm 0.005 \text{ cm}}.$$

The lower and upper bounds for this ratio are

$$\text{Lower Bound}: \quad \frac{d}{h} = \frac{8.48 \text{ cm}}{0.055 \text{ cm}} \approx 150$$
$$\text{Upper Bound}: \quad \frac{d}{h} = \frac{8.52 \text{ cm}}{0.045 \text{ cm}} \approx 190$$

Take the average of these two bounds.

$$\frac{150 + 190}{2} = 170$$

Both bounds can be covered with an uncertainty of 20. Therefore, the ratio is

$$\frac{d}{h} = 170 \pm 20.$$